

Q1: [10 marks]

$$(i) \quad 3x + 4y - z - 2x - 3y + 2z + x + 4z$$
$$= \underline{2x + y + 5z} \quad [2]$$

$$(ii) \quad x(x+4) - x(x-3)$$

the exam question was $x(x+4) - (x+3)$
 $= x^2 + 4x - x - 3$

$$= x^2 + 4x - x^2 + 3x$$
$$= \underline{7x} \quad [2]$$

$$(iii) \quad p^3 q^1 r^1 \times 2p^1 q^4 \times 4q^1 r^2$$
$$= \underline{8 p^4 q^6 r^3} \quad [2]$$

$$iv) \quad \frac{48 p^2 q^3 r}{12 p q r^3} = \underline{4 p q^2 r^4} \quad [2]$$

$$v) \quad \frac{fg}{16h} \times \frac{gh^3}{2fh} \times \frac{32fh^5}{f^4} = \frac{1}{2} \frac{f^2 g^2 h^5}{f^5 h^2}$$

f^8

$$= \frac{1}{2} \underline{f^{-3} g^2 h^3} \quad [2]$$

Q2: [10 marks]

i) $x^2 + 7x + 12 = 0$

$$(x + 3)(x + 4) = 0$$

either

or

$$x + 3 = 0$$

$$x + 4 = 0$$

$$\underline{x = -3}$$

$$\underline{x = -4}$$

[5]

ii) $\frac{(x-3)}{2} = \frac{(2x+1)}{3}$

$$\Rightarrow 3(x-3) = 2(2x+1)$$

$$3x - 9 = 4x + 2$$

$$-9 - 2 = 4x - 3x$$

$$\underline{-11 = x}$$

[5]

Q3: [15 marks]

a i) $5^{x+1} = 6$

$$\log_{10} 5^{x+1} = \log_{10} 6$$

$$(x+1) \log_{10} 5 = \log_{10} 6$$

$$x+1 = \frac{\log_{10} 6}{\log_{10} 5}$$

$$x = \frac{\log_{10} 6}{\log_{10} 5} - 1$$

$$= 0.11(24/5)$$

[6]

$$3(ii) \quad 2 \log x = \log(5x-6)$$

$$\Rightarrow \log x^2 = \log(5x-6)$$

$$\Rightarrow x^2 = 5x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

either

$$x - 2 = 0$$

$$\underline{x = 2}$$

OR

$$x - 3 = 0$$

$$\underline{x = 3}$$

[4]

Q4: [10 marks]

$$\underline{a} = \underline{i} + 2\underline{j} + 3\underline{k} \quad , \quad \underline{b} = \underline{i} + \underline{j} + \underline{k}$$

$$\begin{aligned} (i) \quad \underline{a} \cdot \underline{b} &= (\underline{i} + 2\underline{j} + 3\underline{k}) \cdot (\underline{i} + \underline{j} + \underline{k}) \\ &= (1 \times 1) + (2 \times 1) + (3 \times 1) \\ &= 1 + 2 + 3 \\ &= 6 \end{aligned}$$

$$\begin{aligned} (ii) \quad |\underline{a}| &= \sqrt{1^2 + 2^2 + 3^2} \\ &= \sqrt{1 + 4 + 9} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} |\underline{b}| &= \sqrt{1^2 + 1^2 + 1^2} \\ &= \sqrt{1 + 1 + 1} \\ &= \sqrt{3} \end{aligned}$$

(ii)^{Cont}

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$6 = \sqrt{14} \sqrt{3} \cdot \cos \theta$$

$$\frac{6}{\sqrt{14} \sqrt{3}} = \cos \theta$$

$$\cos^{-1} \left[\frac{6}{\sqrt{14} \sqrt{3}} \right] = \theta$$

$$\underline{22.21^\circ = \theta}$$

[2]

$$(iii) \quad \underline{c} = 2\underline{i} - \underline{j} + 0\underline{k}$$

$$\underline{a} = \underline{i} + 2\underline{j} + 3\underline{k}$$

$$\underline{c} \cdot \underline{a} = (2 \times 1) + (-1 \times 2) + (0 \times 3)$$

$$= 2 - 2 + 0$$

$$= 0$$

[2]

$$\Rightarrow \underline{c}, \underline{a} \text{ are } \perp \text{ as } \cos 90^\circ = 0.$$

$$(iv) \quad \underline{a} = \underline{i} + 2\underline{j} + 3\underline{k}$$

$$\underline{b} = \underline{i} + \underline{j} + \underline{k}$$

$$\underline{d} = \underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\underline{d} = \underline{i} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - \underline{j} \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + \underline{k} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$\underline{d} = \underline{i}(2-3) - \underline{j}(1-3) + \underline{k}(1-2)$$

$$= -\underline{i} + 2\underline{j} - \underline{k}$$

$$\underline{\hat{n}} = \frac{\underline{d}}{|\underline{d}|} = \frac{-\underline{i} + 2\underline{j} - \underline{k}}{\sqrt{(-1)^2 + (2)^2 + (-1)^2}}$$

$$\underline{\hat{n}} = \frac{-\underline{i} + 2\underline{j} - \underline{k}}{\sqrt{6}}$$

[4]

Q5: [15 marks]

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{bmatrix}$$

$$|A| = +3 \begin{vmatrix} -1 & 2 \\ -3 & -4 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix}$$

$$= 3 \{ 4 - (-6) \} - 2 \{ -8 - 2 \} + 1 \{ -6 - (-1) \}$$

$$= 3 \{ 10 \} - 2 \{ -10 \} + 1 \{ -5 \}$$

$$= 30 + 20 - 5$$

$$= \underline{45}$$

Matrix of Co-factors:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$C = \begin{bmatrix} + \begin{vmatrix} -1 & 2 \\ -3 & -4 \end{vmatrix} & - \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} & + \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} \\ - \begin{vmatrix} 2 & 1 \\ -3 & -4 \end{vmatrix} & + \begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix} & - \begin{vmatrix} 3 & 2 \\ 1 & -3 \end{vmatrix} \\ + \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} & - \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} & + \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} 10 & 10 & -5 \\ +5 & -13 & 11 \\ 5 & -4 & -7 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 10 & +5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot C^T$$

$$= \frac{1}{45} \begin{bmatrix} 10 & +5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{bmatrix}$$

Q6: [10 marks]

$$3x + 2y + z = 6$$

$$2x - y + 2z = 3$$

$$x - 3y - 4z = -6$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}$$
$$\underline{A} \underline{x} = \underline{b}$$

$$\Rightarrow \underline{A}^{-1} \underline{A} \underline{x} = \underline{A}^{-1} \underline{b}$$
$$\underline{I} \underline{x} = \underline{A}^{-1} \underline{b}$$
$$\underline{x} = \underline{A}^{-1} \underline{b}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{45} \begin{bmatrix} 10 & +5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{45} [(10 \times 6) + (+5 \times 3) + (5 \times -6)]$$

$$\begin{aligned} x &= \frac{1}{45} [60 + 15 - 30] \\ &= \frac{1}{45} (45) \\ &= \underline{1} \end{aligned}$$

$$y = \frac{1}{45} [(10 \times 6) + (-13 \times 3) + (-4 \times -6)]$$

$$\begin{aligned} &= \frac{1}{45} [60 - 39 + 24] \\ &= \frac{1}{45} (45) \\ &= \underline{1} \end{aligned}$$

$$z = \frac{1}{45} [(-5 \times 6) + (11 \times 3) + (-7 \times -6)]$$

$$= \frac{1}{45} [-30 + 33 + 42]$$

$$= \frac{1}{45} [45]$$

$$= \underline{1}$$

$$\underline{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

Q7: [20 marks]

a i) $y = 4x$

$$\underline{\frac{dy}{dx} = 4}$$

[2]

(ii) $y = \frac{1}{x} + 8x^{1/2}$

$$\frac{dy}{dx} = -1x^{-2} + 8 \cdot \frac{1}{2} x^{-1/2}$$

$$= x^{-2} + 4x^{-1/2}$$

$$= \underline{\frac{1}{x^2} + \frac{4}{\sqrt{x}}}$$

[2]

(iii) $y = x^{-4} + 7x^3$

$$\frac{dy}{dx} = -4x^{-5} + 21x^2$$

$$= \underline{-\frac{4}{x^5} + 21x^2}$$

[2]

(iv) $y = \sin(2x)$

$$\frac{dy}{dx} = \underline{2\cos(2x)}$$

[2]

v) $y = x^3 \cos(x)$ via product Rule.

$$\begin{aligned}\frac{dy}{dx} &= 3x^2 \cos(x) + x^3 (-\sin(x)) \\ &= x^2 [3\cos(x) - x\sin(x)] \quad [3]\end{aligned}$$

vi) $y = (5x+3)^{10}$ (via chain Rule)

$$\begin{aligned}\frac{dy}{dx} &= 10(5x+3)^9 \cdot 5 \\ &= \underline{50(5x+3)^9} \quad [2]\end{aligned}$$

vii) $y = e^x + \ln(x) - 3$

$$\frac{dy}{dx} = e^x + \frac{1}{x} \quad [3]$$

7b $y = \frac{(x-1)^3}{(1+x^2)}$ (via Quotient Rule)

$$\begin{aligned}\frac{dy}{dx} &= \frac{(1+x^2) \cdot 1 - (x-1)^3 \cdot 2x}{(1+x^2)^2} \\ &= \underline{\frac{1+x^2-2x^2+2x}{(1+x^2)^2}}\end{aligned}$$

$$\frac{dy}{dx} = \frac{1 + 2x - x^2}{(1+x^2)^2}$$

when $x=0$, $\frac{dy}{dx} = \frac{1}{(1)^2} = 1$ [4]

Q8: [10 marks]

$$\begin{aligned} \text{i) } \int (2x + 5x^4) dx &= \frac{2x^2}{2} + \frac{5x^5}{5} + C \\ &= \underline{x^2 + x^5 + C} \quad [2] \end{aligned}$$

$$\begin{aligned} \text{ii) } \int (3x^2 + 5e^x) dx &= \frac{3x^3}{3} + 5e^x + C \\ &= \underline{x^3 + 5e^x + C} \quad [2] \end{aligned}$$

$$\text{iii) } \int \left(\frac{1}{x} - 4 \right) dx = \ln(x) - 4x + C \quad [2]$$

$$\begin{aligned} \text{iv) } \int x(3x+4) dx &= \int (3x^2 + 4x) dx \\ &= \frac{3x^3}{3} + \frac{4x^2}{2} + C \\ &= \underline{x^3 + 2x^2 + C} \quad [2] \end{aligned}$$

$$\begin{aligned}
 v) \int_0^1 (4x^3 - 2x) dx &= \left[\frac{4x^4}{4} - \frac{2x^2}{2} \right]_0^1 \\
 &= \left[x^4 - x^2 \right]_0^1 \\
 &= (1^4 - 1^2) - (0^4 - 0^2) \\
 &= 0 - 0 \\
 &= \underline{0} \qquad [2]
 \end{aligned}$$